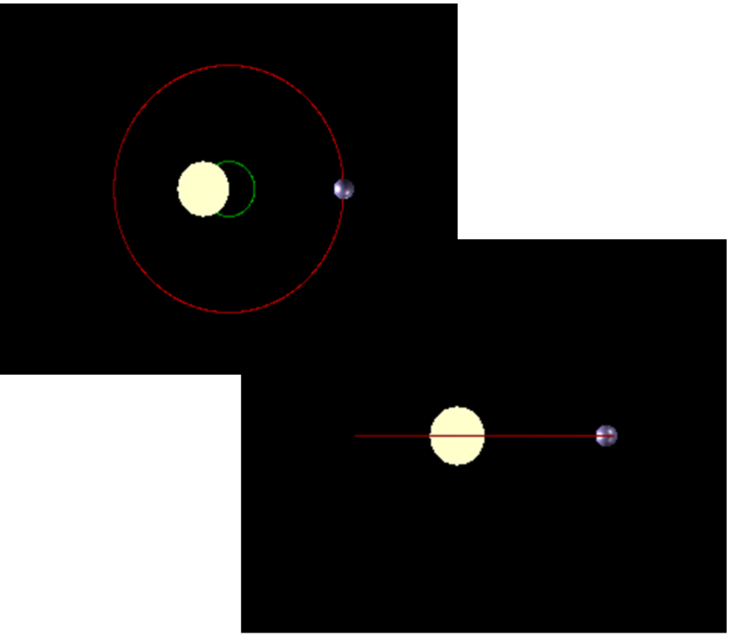
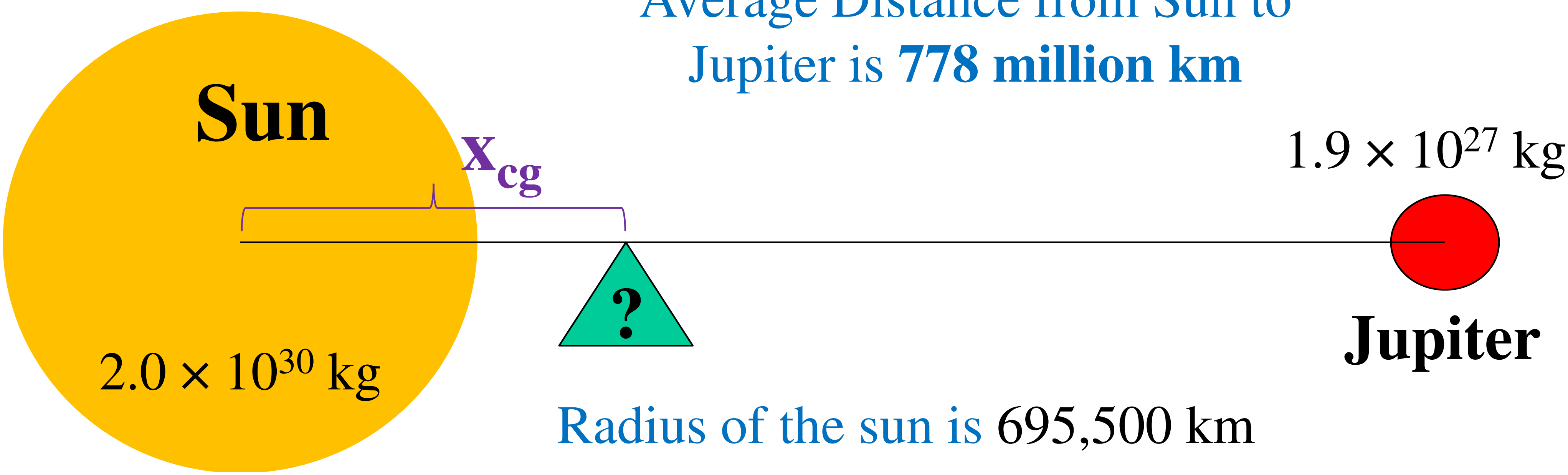


Center of Mass in Astrophysics

Star/planet systems (and other objects) rotate about center of mass

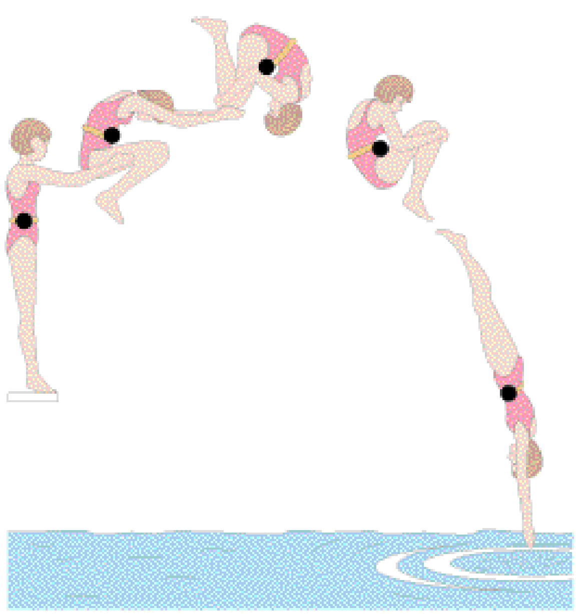
Imagine if our solar system only consisted of the Sun and Jupiter. Where would the center of mass be?

Average Distance from Sun to Jupiter is **778 million km**



Radius of the sun is 695,500 km

A diver can reduce her moment of inertia by a factor of about 3.5 when changing form the straight position to the tuck position. If she makes two rotations in 1.5 s when in the tuck position, what is her angular speed (rev/s) when in the straight position?



How should we approach this problem?

Compare to next problem

Sammy is on Top Chef and Padma tells him to hold a 2.00 kg carton of milk at arm's length. What force F_B must be exerted by the biceps muscle?

What does this have to do with torque or equilibrium?

Everyone stand up. Grab something heavy. Is it harder to hold it straight out or with your elbows bent?

A. Straight out

B. With elbows bent

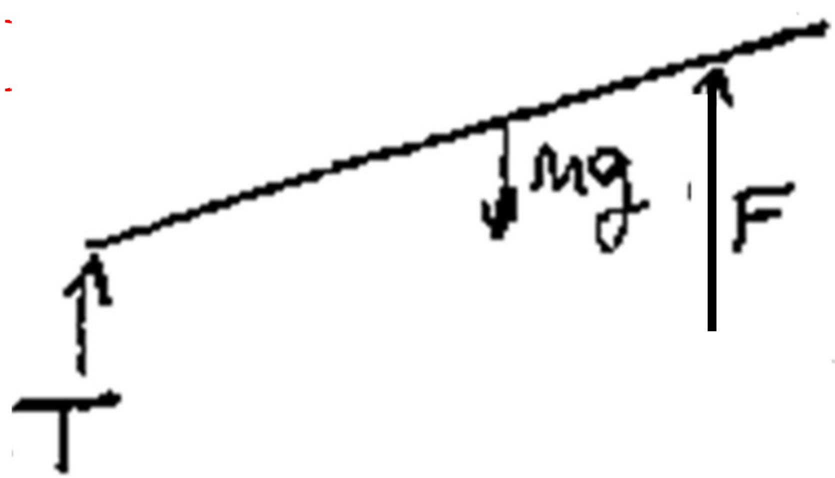
C. Same for both

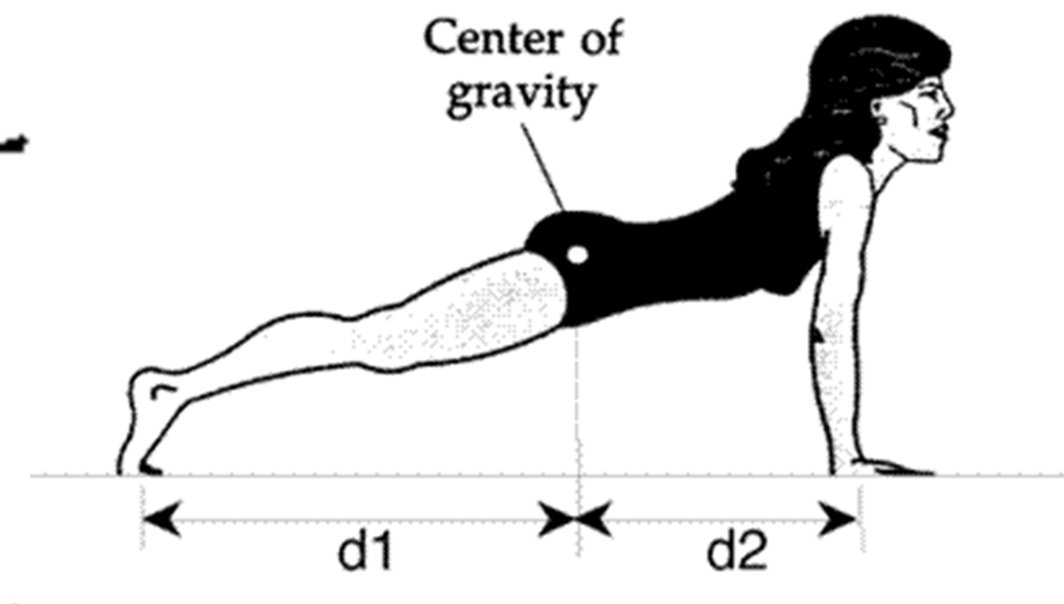
Q106

Mary is about to do a push-up. Her center of gravity lies directly above a point on the floor which is d_1=1.0 *m* from her feet and d_2=0.7 *m* from her hands.

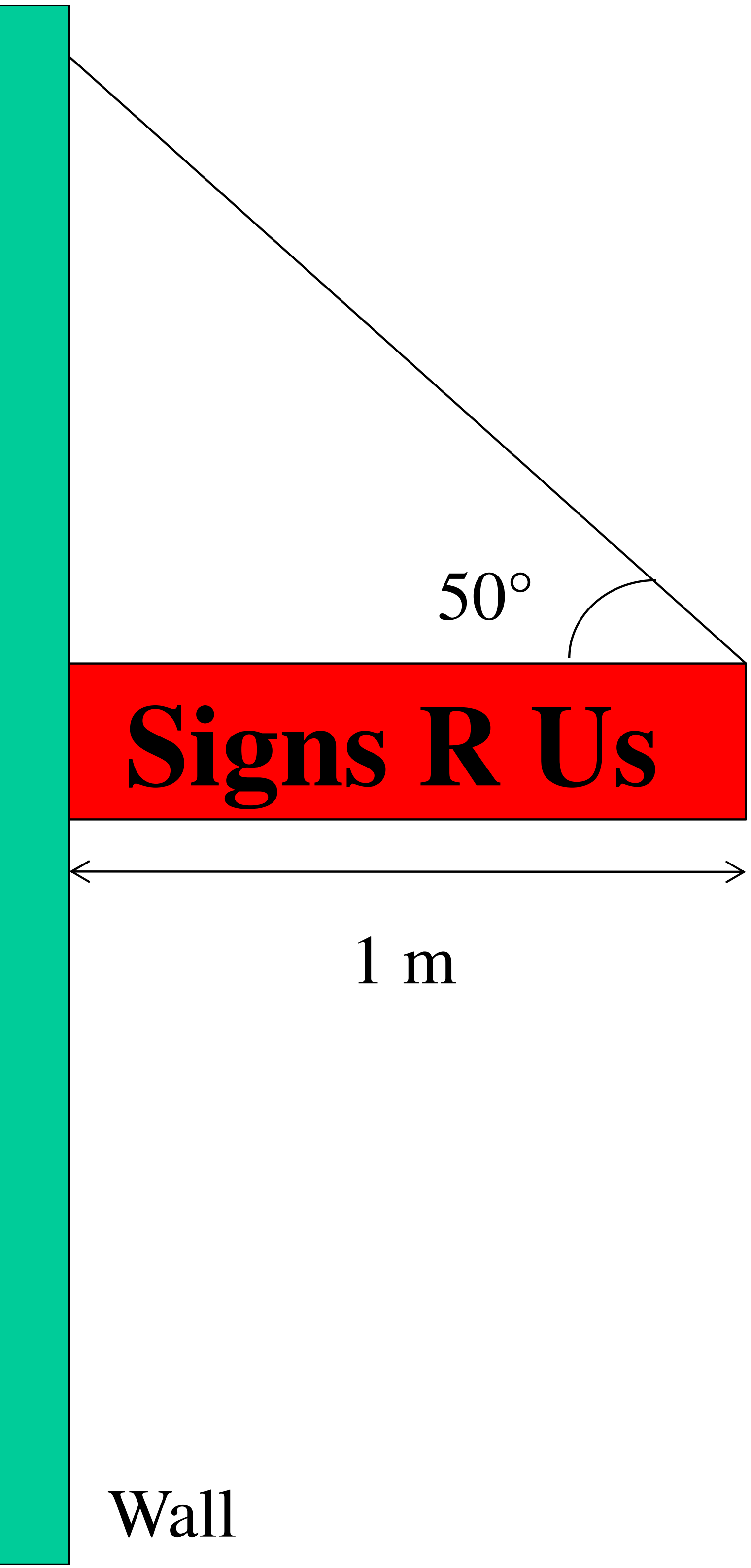
A) If her mass is 50 *kg*, what is the force exerted by the floor on her hands, assuming that she holds this position?

B) What is the force exerted by the floor on her feet?





Signs in Equilibrium



What tension will be needed in the rope to support the sign's 15 kg weight and keep it from falling off the wall?

Find the centripetal acceleration and final rotational kinetic energy of a amusement park 1000 kg centrifuge (r=5 m) that starts from rest and after 5 second is going 3 rad/s. Treat like a hoop ($I=MR^2$).

Relations

Linear Motion	Rotational Motion
Mass m	Moment of Inertia I
Linear velocity **v**	Angular velocity ω
Translational KE $\frac{1}{2}mv^2$	Rotational KE $\frac{1}{2}I\omega^2$
Linear momentum $\mathbf{p} = m\mathbf{v}$	Angular momentum $L = I\omega$
$\vec{F}_{net} = \sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t}$	$\tau_{net} = \sum \tau = \frac{\Delta L}{\Delta t}$

(F=ma)
when m constant

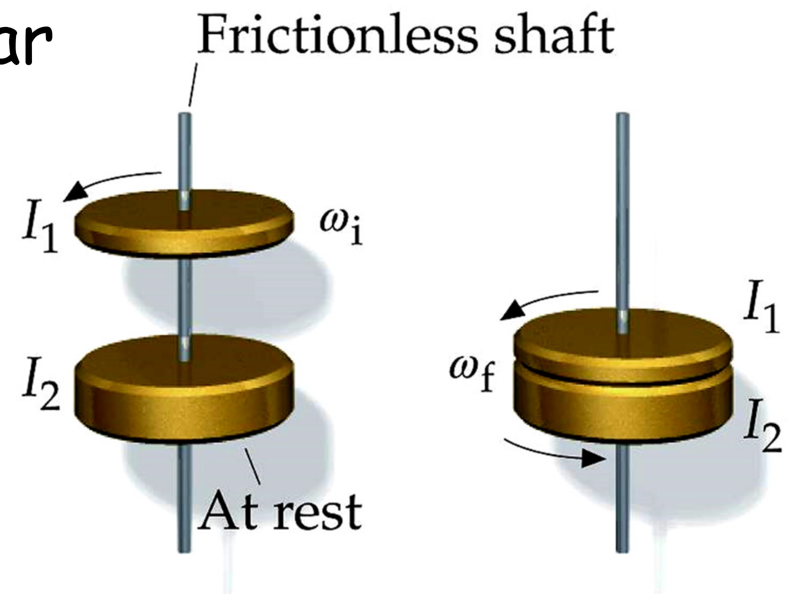
Note: if I is constant, $\frac{\Delta L}{\Delta t} = \frac{I\omega - I\omega_o}{\Delta t} = \frac{I\Delta\omega}{\Delta t} = I\alpha$

I is less likely to be constant than mass

For constant α: $\omega = \omega_o + \alpha t$ $\Delta\theta = \omega_o t + \frac{1}{2}\alpha t^2$ $\omega^2 = \omega_o^2 + 2\alpha\Delta\theta$

Example: A Rotating Disk

Disk 1 is rotating freely and has angular velocity ω_i and moment of inertia I_1 about its symmetry axis, as shown. It drops onto disk 2 of moment of inertia I_2, initially at rest. Because of kinetic friction, the two disks eventually attain a common angular velocity ω_f.



(a) What is ω_f?

(b) What is the ratio of final to initial kinetic energy?

$$L_f = L_i$$

$$(I_1 + I_2)\omega_f = I_1\omega_i$$

$$\omega_f = \frac{I_1}{I_1 + I_2}\omega_i = \boxed{\frac{1}{1+(I_2/I_1)}\omega_i}$$

$$K = \tfrac{1}{2} I\omega^2 = \frac{(I\omega)^2}{2I} = \frac{L^2}{2I}$$

$$\frac{K_f}{K_i} = \left(\frac{L^2}{2I_f}\right) / \left(\frac{L^2}{2I_i}\right) = \frac{I_i}{I_f} = \boxed{\frac{I_1}{I_1 + I_2}}$$

Example: A Stellar Performance

A star of radius R_i = 2.3 x 10^8 m rotates initially with an angular speed of ω_i = 2.4 x 10^{-6} rad/s.

If the star collapses to a neutron star of radius R_f = 20.0 km, what will be its final angular speed ω_f?

$$L_i = L_f \quad \Rightarrow \quad I_i \omega_i = I_f \omega_f$$

$$\omega_f = \left(\frac{I_i}{I_f} \right) \omega_i = \frac{\frac{2}{5} M R_i^2}{\frac{2}{5} M R_f^2} \omega_i = \left(\frac{R_i}{R_f} \right)^2 \omega_i$$

$$= \left[\frac{(2.3 \times 10^8 \text{ m})}{(2.0 \times 10^4 \text{ m})} \right]^2 (2.4 \times 10^{-6} \text{ rad/s}) = 320 \text{ rad/s}$$

$$= 3056 \text{ rpm}$$